

Analytical Geometry

Straight line parallel to the x-axis: $m = 0$
 Straight line parallel to the y-axis: $m = \text{undefined}$

Straight line equation:

$$y = mx + c$$

Gradient formula:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Parallel gradients:

$$m_1 = m_2$$

Perpendicular gradients:

$$m_1 \times m_2 = -1$$

Distance:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Co-linear:

$$m_{AB} = m_{BC} \text{ OR } d_{AB} + d_{BC} = d_{AC}$$

Collinear points A , B and C lie on the same line

Midpoint formula:

$$M(x; y) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

Midpoint Theorem: If two midpoints on adjacent sides of a triangle are joined by a straight line, the line will be parallel to and half the distance of the third side of the triangle.

Converting gradient (m) into angle of inclination (θ)

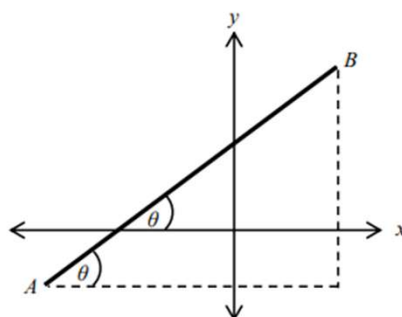
$$m_{AB} = \frac{\Delta y}{\Delta x}$$

and

$$\tan \theta = \frac{o}{a} = \frac{\Delta y}{\Delta x}$$

therefore;

$$\therefore m_{AB} = \tan \theta$$



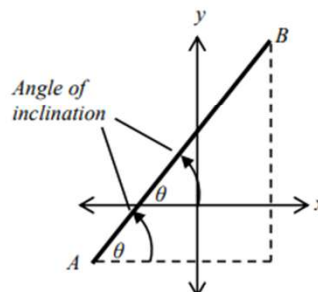
The angle of inclination (θ) is always in relation to a horizontal plane in an anti-clockwise direction.

Positive gradient:

$$m > 0$$

$$\tan^{-1}(m) = \theta$$

The reference angle is equal to the angle of inclination.



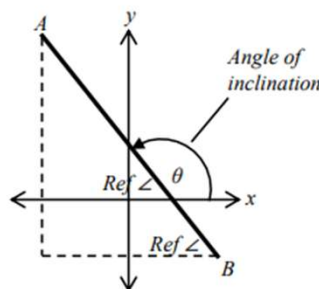
Negative gradient:

$$m < 0$$

$$\tan^{-1}(m) = \text{ref } \angle$$

Angle of inclination:
 $\theta + \text{ref } \angle = 180^\circ$ ($\angle$'s on str. line)

The angle of inclination must be calculated from the reference angle.



Converting a positive gradient into an angle

$$m > 0$$

$$\tan^{-1}(m) = \theta$$

The reference angle is equal to the angle of inclination.

Converting a negative gradient into an angle

$$m < 0$$

$$\tan^{-1}(m) = \text{ref } \angle$$

Angle of inclination:
 $\theta + \text{ref } \angle = 180^\circ$ ($\angle$'s on str. line)

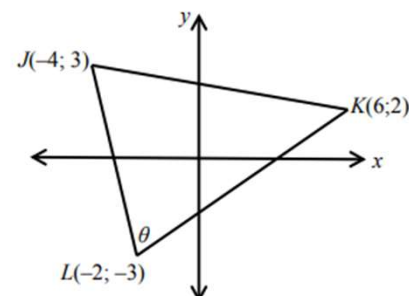
Converting an angle into a gradient

Sub. the ref. $\angle$ into $m = \tan \theta$.

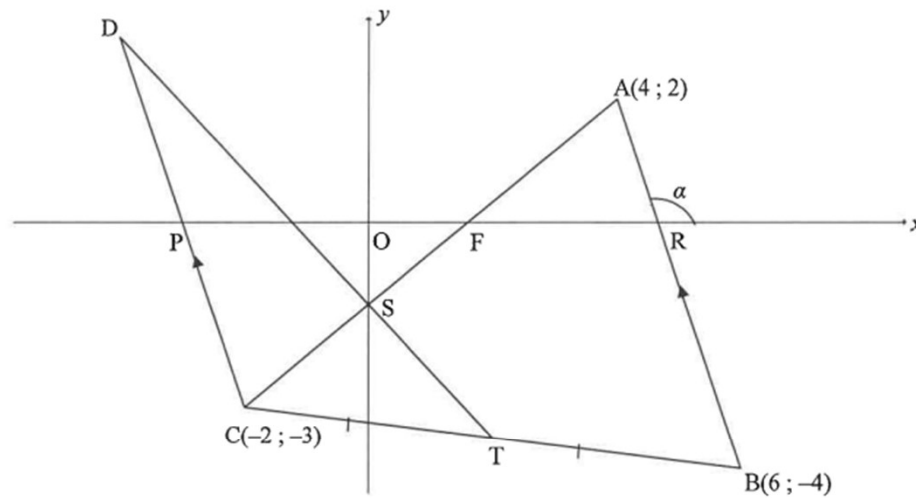
Remember to add the $-$ sign to answers for negative gradients.

Finding an angle that is not in relation to a horizontal plane

Construct a horizontal plane, parallel to the x -axis. This will allow you to use the 'sum of adjacent angles on a straight line' in order to calculate the value of the angle.



In the diagram, $A(4; 2)$, $B(6; -4)$ and $C(-2; -3)$ are vertices of $\triangle ABC$. T is the midpoint of CB . The equation of line AC is $5x - 6y = 8$. The angle of inclination of AB is α . $\triangle DCT$ is drawn such that $CD \parallel BA$. The lines AC and DT intersect at S , the y -intercept of AC . P , F and R are the x -intercepts of DC , AC and AB respectively.



- 3.1 Calculate the:
- 3.1.1 Gradient of AB (2)
 - 3.1.2 Size of α (2)
 - 3.1.3 Coordinates of T (2)
 - 3.1.4 Coordinates of S (2)
- 3.2 Determine the equation of CD in the form $y = mx + c$. (3)
- 3.3 Calculate the:
- 3.3.1 Size of $\hat{DCA}$ (4)
 - 3.3.2 Area of $\triangle POSC$ (5)
- [20]

What is Analytical Geometry?

Analytical Geometry (also called Co-ordinate Geometry) is the study and application of straight line functions, trigonometry and Euclidean Geometry by using points on a Cartesian Plane. In Grade 12 we combine this knowledge and find the equation of circles.

Prior Knowledge:

1. All analytical formulae (distance, midpoint, gradients, and straight line functions)
2. Euclidean Geometry (types of triangles and quadrilaterals, circle geometry theorems)
3. Trigonometry (general applications, sine and cosine rules, sine area rule, double and compound angle identities)

Glossary of Terms:

Concentric

Two or more circles that share the same centre.

Median

Line from the vertex of a triangle to the midpoint of the opposite side.

Centroid

Point of intersection for all the medians.

Altitude

Perpendicular line drawn from a side of a triangle to the opposite vertex.

Orthocentre

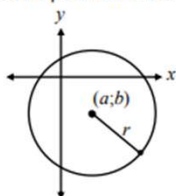
Point of intersection for all the altitudes.

EQUATION OF A CIRCLE

The equation of a circle is given by the equation:

$$(x - a)^2 + (y - b)^2 = r^2,$$

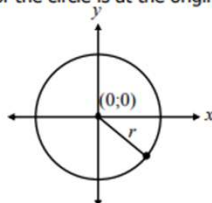
where $(a; b)$ is the centre of the circle, and x and y are coordinates of a point on the circle.



With radius as the subject:

$$r = \sqrt{(x - a)^2 + (y - b)^2}$$

If the centre of the circle is at the origin $(0;0)$:



$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x - 0)^2 + (y - 0)^2 = r^2$$

$$\therefore x^2 + y^2 = r^2$$

OR

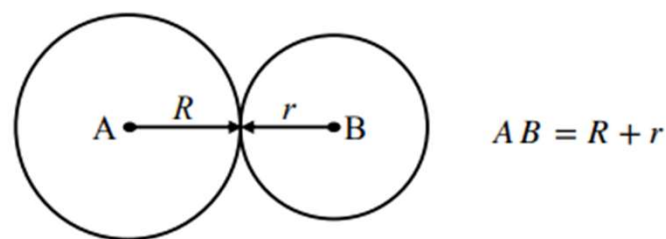
$$r = \sqrt{x^2 + y^2}$$

The standard centre-radius equation can be expanded to give;

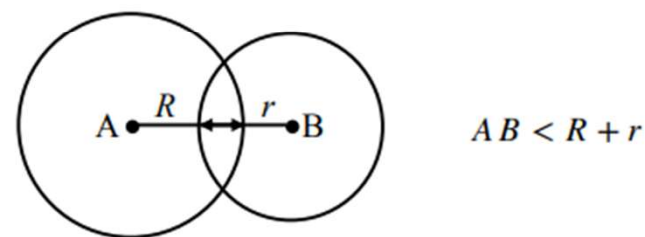
$$Ax^2 + Bx + Cy^2 + Dy + E = 0$$

Given two circles with centres A and B respectively, it can be determined if the circles are:

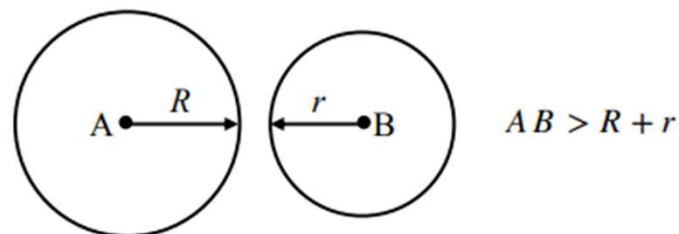
a) Touching externally (one point of intersection)



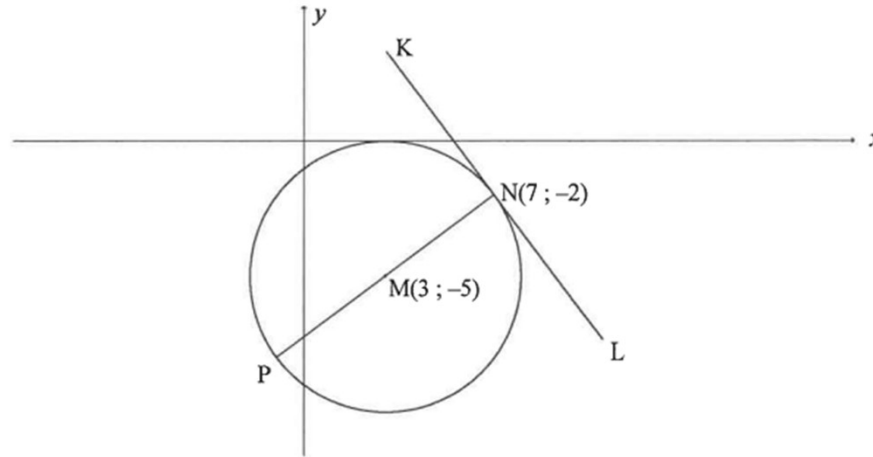
b) Touching internally (Two points of intersection)



c) Not touching at all



In the diagram, $M(3; -5)$ is the centre of the circle having PN as its diameter. KL is a tangent to the circle at $N(7; -2)$.



- 4.1 Calculate the coordinates of P . (2)
- 4.2 Determine the equation of:
- 4.2.1 The circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (3)
- 4.2.2 KL in the form $y = mx + c$ (5)
- 4.3 For which values of k will $y = -\frac{4}{3}x + k$ be a secant to the circle? (4)
- 4.4 Points $A(t; t)$ and B are not shown on the diagram.
- From point A , another tangent is drawn to touch the circle with centre M at B .
- 4.4.1 Show that the length of tangent AB is given by $\sqrt{2t^2 + 4t + 9}$. (2)
- 4.4.2 Determine the minimum length of AB . (4)
- [20]